Validating the Lattice Boltzmann Method for the Characterization of Impedances in Pipes

Andrey R. da Silva*, **Gary Scavone**

[1]Computational Acoustic Modeling Laboratory – Music Technology Area - McGill University
555 Sherbrooke Street West – H3A 1E3 Montreal, QC

andrey.dasilva@mail.mcgill.ca, gary@music.mcgill.ca

Abstract. *This paper proposes the application of the Lattice-Boltzmann method (LBM) for the characterization of the acoustical impedance and sound radiation properties of wind instruments. The validation of the method is achieved by simulating the radiation of an open unflanged pipe with LBM and comparing the impedance related data and radiation results with those obtained through the exact solution derived by Levine and Schwinger for the same model.*

1. Introduction

One important issue in sound synthesis by physical modeling is to create simplified representations of the dynamic behavior of a musical instrument, trying to take into account only the parameters that play a significant role on the auditory perception so that the model can be efficiently reproduced in real time.

In the case of the wind instruments, these parameters can be well described in terms of acoustic impedances. The knowledge of the instrument impedances allows solutions to be found for the sound fields inside the instrument bore and the sound power transmitted into the surrounding air. In other words, it is essential to describe the interaction between the instrument and other systems (lips, vocal tract, environment, etc.) and also to represent the way the instrument radiates sound as a function of frequency and space. For the purpose of sound synthesis, the impedance information can subsequentlly be represented by digital filters as described by [Scavone, 1999]. The determination of the impedance is not a straightforward task, however. The existing analytical expressions are restricted to systems with simple geometries. In the other hand, experimental procedures are usually frequency limited and have other geometrical issues. Furthermore, it relies on the existence of a real prototype. Alternatively, the acoustic impedance of more complex systems can be derived numerically.

This paper presents the validation of Lattice Boltzmann Method (LBM) for the numerical determination of the acoustic impedance of a simple pipe. The validation is done by comparing the numerical results obtained from a Lattice Boltzmann model of an open pipe with the exact theoretical results for the radiation and impedance of an unflanged open pipe derived by [Levine and Schwinger, 1948].

2. A Brief description of the Lattice Boltzmann Method

The fundamental theory of the LBM has been thoroughly described in many publications, e.g., [Succi, 2001] and [Gladrow, 2000]. Although relatively new, LBM has

*Supported by CAPES.

been extensively studied in many disciplines. However, its use in acoustics has been explored by only a limited number of authors. [Buick et al., 1998] has studied the the propagation of waves considering dissipative effects for different boundary conditions and [Buick et al., 2000] have applied it on the simulation of shock wave fronts. [Skordos, 1995] simulated the mechanism of sound generation of an organ pipe by using a 3D LB model. [Lallemand and Luo, 2003] studied the propagation of waves by using different lattice models.

The main idea of the method is to create a discretized version the Boltzmann equation, which describes the fluid at the microdynamic level by a distribution function. Once discretized, the equation can be approximated numerically. The discretization is implemented by the use of a lattice grid where each lattice represents a limited number of velocity states that can be taken by a particle in the fluid. At each discrete time interval all the particles of a domain are found at lattice sites where they collide and exchange momentum according to their previous values. The LBM code used in our simulation was implemented in Matlab and uses a two dimensional squared lattice model known as D2Q9 where each particle in a fluid can assume 9 different velocity directions.

3. Method Validation

The validation of the method was based on the comparison of results obtained by the LB simulation and the theoretical results obtained by [Levine and Schwinger, 1948] for an open-end unflanged pipe, which had also been experimentally validated by [Ando, 1968].

3.1. Model Setup

The model was built with a grid of 500 X 500 lattices, assuming a discretization N = 1000 lattices per meter. The space step between neighbor grid points, δx, was, therefore, equal to 1 mm. The criteria proposed by [Wilde, 2004], which suggests that the highest wavelength being analyzed should be resolved with at least 12 lattices was adopted, so that a numerical error for the phase velocity of less than to 1 per cent could be guaranteed for a frequency resolution up to 28 kHz.

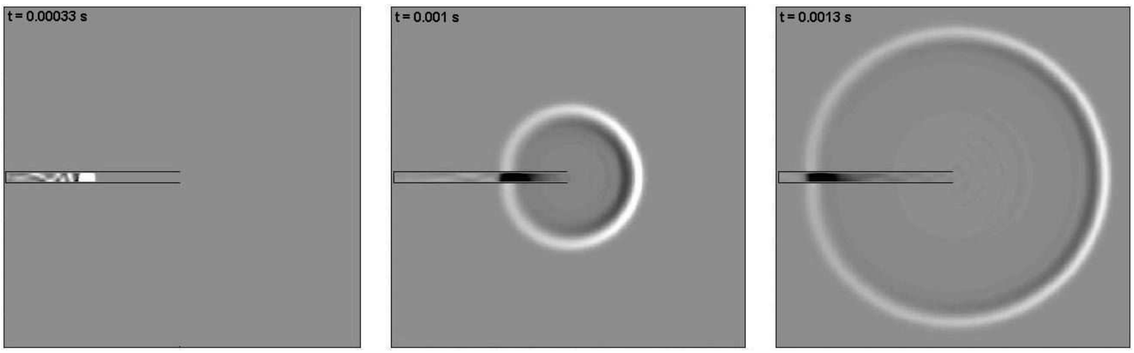

Figure 1: Sound radiation from the pipe model at different time steps

An axisymmetric model of the pipe was simulated by implementing a bounce-back scheme in which the walls of the pipe were considered to be dry lattices. This means that, during the evolution process, the particles propagating in the direction of the walls would be reflected back in the opposite direction conserving the same momentum. The pipe was modeled to be 25 cm long with an inner diameter of 2.5 cm. The lattices at the outer boundaries of the grid were set to have an anechoic behavior so that all the acoustic energy reaching them would be totally absorbed instead of being reflected back into the grid domain. The pipe was excited with an impulse at the closed end. The impulse was generated by applying a density disturbance equal to 0.1 per cent higher than the fluid density at $t = 0s$. It was expected that the density disturbance would propagate as a wavefront along the pipe until it reached the pipe open end where it should be partially reflected back into the pipe and partially radiated to the outside. Figure 1 depicts the axisymmetric model of the pipe within the lattice grid with anechoic boundaries.

Two probes were placed at the open end of the pipe model to acquire pressure, $p(t)$, and velocity, $v(t)$, at every time step in order to determine the output impedance of the pipe and, consequently, the reflectance. Moreover, 38 probes were displaced in a semi-circle centered at the pipe's open end with a radius of 12.5 cm to acquire the radiated pressure.

3.2. Impedance Results

The complex values of the particle velocity, $V(\omega)$, and pressure, $P(\omega)$, in the frequency domain were obtained by a fast Fourier transform of the time-domain signals acquired at the pipe's open end. These results were used to derive the complex impedance, $Z(\omega)$, and the reflectance magnitude, $R(\omega)$. Figures 2 depicts the comparison between the results for the normalized impedance and reflectance acquired numerically and the theoretical results of Levine and Schwinger plotted as a function of *ka*, where *a* is the radius of the pipe, $k = \frac{\omega}{c}$ is the wavenumber, where ω is the angular frequency, and *c* the speed of sound in the fluid.

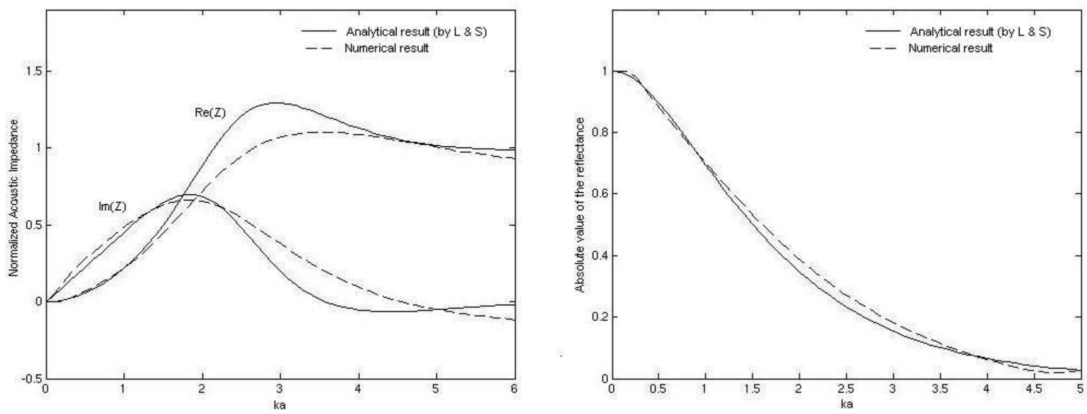

Figure 2: Normalized impedance and reflectance as functions of ka

The results for the impedance (Figure 2 on the left) agree well with a discrepancy of less than 5 per cent for values of $ka < 2$, which corresponds to a linear frequency equal to 8.7 kHz for a pipe of 1.25 cm radius at an air temperature of $20\,^{\circ}\mathrm{C}$ and pressure equal to 1 atmosphere. The plots of the absolute value of the reflectance (Figure 2 on the right) show a good agreement for the whole range up to $ka = 5$. The maximum discrepancy can be found around $ka = 2.3$, which corresponds to a value of 4 per cent higher than the theoretical result.

3.3. Radiation Results

The results for the sound radiation were obtained from the data acquired by the probes placed at the semi-circumference around the pipe's open end and compared with the theoretical results. Figure 3 depicts the polar plots of the sound radiation in the form of power gain for three different values of *ka*. A good agreement can be seen with the results predicted theoretically, especially when $ka > 2$. The discrepancy for lower values of *ka* may be attributed to the existence of an acoustic near field between the pipe's open end and the acquiring probes.

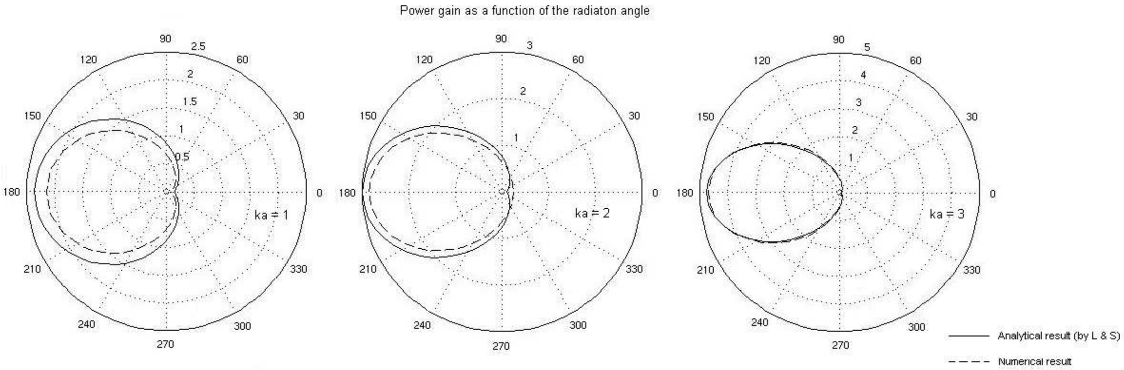

Figure 3: Radiation power gain as a function of angle for different values of ka

4. Conclusions

The motivation of this study was to validate LBM as a tool for determining impedance and radiation properties of wind instruments. The results acquired by the LBM have shown that it is a potential method to be applied on the determination of acoustic impedance and radiation properties of complex geometries. The most significant advantage over the classical continuum methods (Finite Elements, Finite Volumes, etc.) consists on its simplicity of implementation and facility of simulating wave propagation within complicated boundary conditions in the time-domain.

References

Ando, Y. (1968). Experimental Study of the Pressure Directivity and the Acoustic Centre of the Circular Pipe Horn Loud Speaker. *Acustica*, 20(1):366–369.

Buick, J. M., Buckley, C. L., Greated, C. A., and Gilbert, J. (2000). Lattice Boltzman BGK Simulation of Nonlinear Sound Waves. *J. Phys. A*, 33:3917–3928.

Buick, J. M., Greated, C. A., and Campbell, D. M. (1998). Lattice BGK Simulation of Sound Waves. *Europhysics Letters*, 43:235–240.

Gladrow, D. W. (2000). *Lattice-gas Cellular Automata and Lattice-Boltzmann Models*. Springer-Verlag Berlin, 2nd edition.

Lallemand, P. and Luo, L. S. (2003). Theory of the Lattice-Boltzmann Method: Acoustic and Thermal Properties. *Europhysics Letters*, 68(3):150–175.

Levine, H. and Schwinger, J. (1948). On the Radiation of Sound from an Unflanged Circular Pipe. *Physical Review*, 73(4):383–406.

Scavone, G. P. (1999). Modeling Wind Instrument Sound Radiation using Digital Waveguides. *Proceedings of ICMC 1999*, pages 355–358.

Skordos, P. (1995). Modeling Flue Organ Pipes: Subsonic Flow, Lattice-Boltzann and Parallel Distributed Computers, PhD Thesis, MIT.

Succi, S. (2001). *The Lattice Boltzmann Equation for Fluidynamics and Beyond*. Oxford University Press, 1st edition.

Wilde, A. (2004). Application of the Lattice-Boltzmann Method in Flow Acoustics. *SWING Workshop*, Aachen.