Images from the Aperiodic Time

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Abstract

A diffraction image in solid state physics is a measure of the order inside a distribution of points (waves). The image is related with the Fourier transform of the distribution. Some 1D aperiodic distributions have discrete Fourier components. We use this models in order to structureate the time in a non periodic way with discrete frequency spectrum. The Fibonacci sequence illustrates this approach.

1. Introduction.

The scientific analogy has been very fertile in the evolution of musical thought. In particular Mathematics and Physics are among the disciplines more influent. A composer who wants to learn about a scientific model will go beyond it in order to show his own creative view. But that knowledge before musical experience may open new expression possibilities that can not be anticipated by intuition.

A musical theory must be linked to a theory of the musical time. In this work we want to show how a new type of discrete structures introduced and extensively studied in the last decade in the domain of physical sciences can be used in order to structuralate the time in a non periodic way. The order contained in the temporal structure (rhythm) is reflected in the frequency space (harmonic fields) through the Fourier transform.

2. The continuum pitch-durations-forms.

If we take a duration defined by two impulsions, we repeat it and then the duration is decreased until it lasts 1/16 of second, the impulsions can be heard separately. But if we continue decreasing the duration until 1/32 we begin to hear pitch (B5 approximately).

With this experience (Stockhausen 1960) it can be seen how the same basic process is behind our perception of duration and pitch. On the other hand, durations higher than, say, 8 seconds are related with the form and its articulations.

A first attempt to extend in a large scale what happens inside the sound is to consider the very well known decomposition of a periodic function in a Fourier series. Let’s take the following periodic function

\[ M_a(t) = \sum_{n=-\infty}^{\infty} \delta(t - na) \]

where the Dirac delta-function \( \delta(x) \) has the properties: \( \delta(x) = 0 \) unless \( x = 0 \), \( \delta(0) = \infty \). The function \( M_a(t) \) represents a periodic structure in one dimension with period of length \( a \). If \( t \) denotes the time (in seconds) then the Fourier transform of \( M_a(t) \) is proportional to \( M_\omega(\omega) \) where \( \omega \) denotes the frequency (in Hertz). For instance if we take \( a = \frac{1}{5} \) we get the harmonic series:

\[ C_5, C_3, C_5, C_4, C_5, C_4, B_5, C_6, C_5, C_5, C_4, B_5, C_6, B_5, C_7, \ldots \]

all of them with the same intensity which is measured by the proportionality factor of the function \( M_\omega(\omega) \).

Timbre is generated by summing up harmonics: fragmentation into microdurations of a macroduration defined by the fundamental. If we take timbre as a model for the musical form we can think in a musical piece generated by fragmentation of a global duration. This approach allows to structuralate the musical time in a continuum pitch-durations-forms.

But if a function is not periodic (representing a timbre of musical interest) it doesn’t have in principle a Fourier series decomposition and it is necessary to use the Fourier integral which contains the continuum of frequencies and not only integer multiples of a fundamental frequency. This decomposition can not be incorporated into the musical writing which belongs in essence to a discrete domain.

In the next section we study an example of an aperiodic discrete time structure with discrete frequency spectrum.

3. The Fibonacci time structure.

Aperiodic systems are in some place between order and disorder. A relevant 1D example is the Fibonacci chain with a distribution of points along the real line according to:

\[ x_N = N + \alpha + \frac{1}{\phi} \left[ x + \beta \right] \phi \]

where \( \lfloor x \rfloor \) is the greatest integer less than \( x \), \( \phi = \frac{1 + \sqrt{5}}{2} \) is the golden number, \( \alpha \) and \( \beta \) are arbitrary real numbers, and \( N \) is a non-negative integer. This equation describes a sequence of points such that the interval between two consecutive points can be only of two types: \( L = \phi \) and \( S = 1 \) which appear in a non periodic sequence where the ratio of the number of \( L \)-intervals to the number of \( S \)-intervals is also equals to \( \phi \).

An equivalent way to define the Fibonacci sequence is through a formal grammar. Consider the alphabet \( \{ L, S \} \), the production rules \( L \rightarrow LS, S \rightarrow L \) and the axiom \( L \). The language consists in the words \( L, LS, LS, LS, LS, LS, LS, LS, \ldots \).

The Fibonacci chain can be used to structuralate the musical time in a non periodic way. Although the golden number is not rational we can take rational approximants for it. If we define the Fibonacci numbers \( F_n \) as \( F_{n+1} = F_n + F_{n-1} \) with \( F_0 = F_1 = 1 \) it is very well known that \( \frac{F_n}{\phi^n} \to \phi \) when \( n \to \infty \). The sequence \( 1, \frac{1}{\phi}, \frac{1}{\phi^2}, \frac{1}{\phi^3}, \ldots \) can be used to generate aperiodic rhythms with two rhythmic units \( L \) and \( S \) in a rational ratio.

The spectrum of a sequence of points in a line is a sum of discrete and continuous components. The discrete component indicates order, the continuous component disorder. The Fibonacci sequence has only a discrete part and can be computed with the help of the golden number \( \phi \) and two integers \( p \) and \( q \) through the following expression (Levine...
and Steinhardt 1986):

\[ \omega_\alpha = \frac{2\pi}{1 + \frac{1}{\phi^2}} \left[ p + \frac{q}{\phi} \right] \]

and with a different intensity for each component. If we take \( X = 2\pi p - \omega_\alpha / \phi \) then the intensity is proportional to

\[ A \sin^2 \frac{X}{2} \]

A pitch defined by \( p \) and \( q \) is more intense if \( \phi q - p \) is small or \( p/q \) close to \( \phi \), that is when \( (p, q) \) are successive Fibonacci integers \( (F_n, F_{n-1}) \). Outside this sequence the intensities decrease strongly. If we distribute the pitches more intense according with the intensity the following harmonic fields are obtained \( (p, q = 1, 2, \ldots, 20) \), and the frequencies are scaled by a factor ten:

\[ \text{pp:} \{D_2\}; \text{p:} \{A_3, C_4, G_5, C_6\}; \text{mp:} \{F_4, B_5, F_5\}; \text{mf:} \{G_5, D_3, B_6\}; \]

\[ f: \{F_6, G_4, A_5, E_6, G_5\}; \text{ff:} \{D_2, B_3, E_4, B_5, D_5, F_6, A_6, C_6, D_6\} \]

4. Conclusion.
The Fibonacci sequence is an example of the great variety of temporal structures we can get by means of aperiodic systems in 1D. We can think in the pairs intensity-pitch as the Fourier spectrum of aperiodic rhythms generated automatically. These sequences have another interesting property: they are self-similar. There exists a transformation in which each interval is subdivided into pieces that can rejoin to form a new sequence with all intervals scaled down by a factor \( \phi \). This hierarchy can be used in order to articulate the global musical form.

Models based on aperiodic systems have been used by the author in works like Moradas for organ, Nocturno for soprano and ensemble, Imágenes for two pianos and others.

References.

Stockhausen K. (1963) ...wie die Zeit vergeht... Texte zur elektronischen und instrumentalen Musik. Bd.I. Dumont Buchverlag, Köln.

THE NECESSITY OF COMPOSING WITH LIVE-ELECTRONICS
A short account of the piece "Gegensätze (gegenellig)" and of the hardware (AUDIACSYSTEM) used to produce the real-time processes on it.

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ABSTRACT
The aim of this paper is to speak about my piece -Gegensätze (gegenellig) (Contraries (reciprocally)) for alto flute, 4 Channel-tape and live electronics (1994): making an account of how and why the work was conceived. The hardware-and-the-software environments which are responsible for the real-time processes (AUDIACSYSTEM, a project carried on by the ICEM (Institut for Computer music and electronic Media) at the Folkwang Hochschule-Essen and by the company Micro-Control GmbH & Co KG, both in Germany) will be described. Finally some examples and passages of the piece will be explained.

"CONTRARIES "

"Gegensätze (gegenellig)" was the result of an idea that I have had for a long time: to compose a piece in which contraries should be shown not only against each other (in a negative way), but also that they could be able to build some kind of unity by creating something completely new, constructive and positive.

My first problem was how to put this into music without using a text about the subject. At the beginning I simply wanted to make a contrast between a normal instrument and a prerecorded tape, but it didn't seem like being the solution to the problem because it could actually show only the contraries themselves but not the reciprocal action of both elements. The instrument should make with the electronic something really new and this should happen in real time and not with recorded material. That was the reason why I first started to work on the tape itself, making some work with two Yamaha synthesizers (TX 802 - TG77) that shouldn't have any relation with normal instruments. I composed then a previous piece for stereo-tape alone, from which I took the materials for the definitive version of the work. When the tape materials were selected, I knew already that the instrument should have to be a very soft one, on the election was that of an alto flute. How should then the "reciprocal action " look like? I was now pretty sure that it should be performed with live-electronics. This decision conducted me to the next problem: what type of live-electronics did I really want and much further, which kind of system should I use? There are basically two ways of working with live-electronics: on one side, those whose aim is to create a new conception of how the live instruments could be projected into a particular space or room, normally using only echoes and delay lines; on the other side, the more complicated ones, in which the sound will be actually processed in real-time (through FM, AM, filters, envelope generators, envelope-followers, transpositions, etc.) up to the point in which the instrument itself could be no longer recognizable. At the ICEM of the Folkwang Hochschule in Essen (Germany), there's no IRCAM board, but there's a completely different project, which has been carried on since eight years at the ICEM by a group of german composers and engineers. This project is the AUDIACSYSTEM, about which I shall speak later in this lecture.

Once I had already got the three Instrumental groups (alto flute, 4 channel-tape and the 4 channel live-electronics), I wanted to prosecute composing each parameters (from the micro-up to the macro-structures) with the same concepts of THESIS-ANTITHESIS working together to create something new, so that at any point of the piece the main idea could be shown. For this purpose, I've chosen very empirically two principles which are opposite to each other: a 'single-principle' and a 'totality-principle'. Both principles should have to be